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Quantum Optics
Winter semester 2018/2019 - Exercise sheet 9
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Problem 1: Characterization of non-classical states of light.

Show that the radiation field state which is a linear superposition of the vacuum state and a single photon state,

$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle,$$

with $a_0, a_1 \in \mathbb{C}$, is a non-classical state.

Problem 2: Second-order correlation function and density matrix.

Consider the state described by the density operator $\hat{\rho} = N\hat{a}^{\dagger m} e^{-\kappa\hat{a}^\dagger\hat{a}}\hat{a}^m$, where N is a normalization constant.
a) Show that it goes over to a Fock state in the limit $\kappa \to \infty$ and to a thermal state in the limit $\kappa \to 0$. HINT: check the matrix elements ρ_{mn} for the respective states. For the thermal state limit, consider also $n \gg 1$.
b) Find $g^{(2)}$ for this state and show that the photon statistics is sub-Poissonian if $\bar{n} < \sqrt{m/(m+1)}$, where $\bar{n} = [\exp(\kappa) - 1]^{-1}$.

Problem 3: Spatial properties of $g^{(2)}$.

Consider a field with a single photon in each of the two modes k, k' with the same frequency ($k = k'$), $|1_k 1_{k'}\rangle = \hat{a}_k^\dagger \hat{a}_{k'}^\dagger|0\rangle$. Show that the (unnormalized) second order correlation function for this field is

$$G^{(2)}(\mathbf{r}, \mathbf{r}'; \tau = 0) = 2\mathcal{E}_{\mathbf{k}}^4[1 + \cos((\mathbf{k} - \mathbf{k}') \cdot (\mathbf{r} - \mathbf{r}'))].$$

Problem 4: Second-order correlation function and non-classicality.

For the squeezed coherent state $\hat{D}(\alpha)\hat{S}(\xi)|0\rangle = |\alpha, \xi\rangle$, show that $g^{(2)}(\tau)$ is given by

$$g^{(2)}(\tau) = 1 + \frac{|\alpha|^2[\cosh(2|\xi|) - \sinh(2|\xi|)\cos(2\theta - \phi) - 1] + \sinh^2(|\xi|)\cosh(2|\xi|)}{(|\alpha|^2 + \sinh^2(|\xi|))^2},$$

where $\alpha = |\alpha|e^{i\theta}$ and $\xi = e^{i\phi}|\xi|$.

b) Show which conditions $|\alpha|$ should fulfill (for fixed values of $|\xi|$, θ and ϕ) for this state to have sub-Poissonian statistics. How does the phase $\Omega = 2\theta - \phi$ influence $g^{(2)}(\tau)$?
c) Estimate the minimum value of $g^{(2)}(\tau)$ in the space of possible values of $|\xi|$, $|\alpha|$ and Ω.